1. Find at least one polynomial \( P(x) \) of degree 2001 such that \( P(x) + P(1-x) = 1 \) holds for all real numbers \( x \).

2. At the end of the school year it became clear that for any arbitrarily chosen group of no less than 5 students, 80% of the marks “F” (failed) received by this group were given to no more than 20% of the students in the group. Prove that at least \( 3/4 \) of all “F” marks were given to the same student.

3. Let \( AH_A, BH_B \) and \( CH_C \) be the altitudes of triangle \( ABC \). Prove that the triangle whose vertices are the intersection points of the altitudes of \( AH_BH_C, BH_AH_C \) and \( CH_AH_B \) is congruent to \( H_AH_BHC \).

4. There are two matrices \( A \) and \( B \) of size \( m \times n \) each filled only by “0” s and “1” ’s. It is given that along any row or column its elements do not decrease (from left to right and from top to bottom). It is also given that the numbers of “1” s in both matrices are equal and for any \( k = 1, \ldots, m \) the sum of the elements in the top \( k \) rows of the matrix \( A \) is no less than that of the matrix \( B \). Prove for any \( l = 1, \ldots, n \) the sum of the elements in the left \( l \) columns of the matrix \( A \) is no greater than that of the matrix \( B \).

5. In a chess tournament, every participant played with each other exactly once, receiving 1 point for a win, 1/2 for a draw and 0 for a loss.

   (a) Is it possible that for every player \( P \), the sum of points of the players who were beaten by \( P \) is greater than the sum of points of the players who beat \( P \)?

   (b) Is it possible that for every player \( P \), the first sum is less than the second one?

6. Let \( P(x) = c_n x^n + c_{n-1} x^{n-1} + \cdots + c_0 \) be a polynomial with integer coefficients. Suppose that \( r \) is a rational number such that \( P(r) = 0 \). Show that the \( n \) numbers

   \[ c_n r, c_n r^2 + c_{n-1} r, c_n r^3 + c_{n-1} r^2 + c_{n-2} r, \ldots, c_n r^n + c_{n-1} r^{n-1} + \cdots + c_1 r \]

   are integers.

7. Determine all real numbers \( a > 0 \) for which there exists a nonnegative continuous function \( f(x) \) defined on \([0, a]\) with the property that the region

   \[ R = \{(x, y); 0 \leq x \leq a, 0 \leq y \leq f(x)\} \]

   has perimeter \( k \) units and area \( k \) square units for some real number \( k \).
8. Let \( n \) be a positive integer, \( n \geq 2 \), and put \( \theta = 2\pi/n \). Define points \( P_k = (k, 0) \) in the \( xy \)-plane, for \( k = 1, 2, \ldots, n \). Let \( R_k \) be the map that rotates the plane counterclockwise by the angle \( \theta \) about the point \( P_k \). Let \( R \) denote the map obtained by applying, in order, \( R_1 \), then \( R_2, \ldots, \) then \( R_n \). For an arbitrary point \((x, y)\), find, and simplify, the coordinates of \( R(x, y) \).

9. Evaluate
\[
\lim_{x \to 1^-} \prod_{n=0}^{\infty} \left( \frac{1 + x^{n+1}}{1 + x^n} \right)^{x^n}.
\]

10. Let \( \mathcal{A} \) be a non-empty set of positive integers, and let \( N(x) \) denote the number of elements of \( \mathcal{A} \) not exceeding \( x \). Let \( \mathcal{B} \) denote the set of positive integers \( b \) that can be written in the form \( b = a - a' \) with \( a \in \mathcal{A} \) and \( a' \in \mathcal{A} \). Let \( b_1 < b_2 < \cdots \) be the members of \( \mathcal{B} \), listed in increasing order. Show that if the sequence \( b_{i+1} - b_i \) is unbounded, then
\[
\lim_{x \to \infty} \frac{N(x)}{x} = 0.
\]